Evaluation of Stress Intensity Factor for A Partially Patched Crack Using an Approximate Weight Function

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A cracked plate with a patch bonded on one side was treated with a crack-bridging model using weight function: assuming continuous distribution of springs acting between th crack surfaces, the stress intensity factor of the patched crack was numerically obtained. Especially in the case of a patched crack subjected to residual non-uniform stress, the stress intensity factor was easily with the corresponding approximate weight function. This paper presented the stress intensity factors for a crack partially patched within a finite plate or a patched crack initiated from a notch.

Key Words: Crack-bridging Model, Stress Intensity Factor, Weight Function, Patched Crack, Notch

Nomenclature

a	: Crack length
$g(x_i, x_j)$	: Influence function
k	: Constraint factor
K	: Stress intensity factor
$m(x, a)$	: Weight function
S	: Stiffness ratio between plate and reinforcement
$u(x)$	: Crack surface displacement
$\sigma(x)$	: Applied stress distribution

1. Introduction

The repair process, using adhesively bonded composite materials of high stiffness, provides efficient and cost-effective repairs for cracked aircraft components. Compared to mechanical methods such as riveting or bolting, adhesive bonding provides more uniform and efficient load transfer into the patch from the cracked component to introduce much lower stress concentrations. Especially, the recent advances in adhesive bonding techniques and composite materials have sparked new interest in repairing cracked aircraft components with bonded composite patches.

On the other hand, the crack-bridging model, assuming a continuous distribution of springs between the crack faces, has been successfully used to calculate the stress intensity factor for a patched crack within an infinite plate (Rose, 1988, 1987). Additionally, the stress intensity factor for a center crack subjected to mixed mode loading within an infinite plate was also obtained from the crack-bridging model (Wang and Rose, 1998, 1999; Kim and Lee, 2000). To obtain the stress intensity factor, the weight function method has been a very useful and versatile method of calculating stress intensity factors for cracks subjected to non-uniform stress fields such as residual stress or thermal loading (Bueckner, 1970; Rice, 1972). Recently, the stress intensity factor

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for a center crack with a patch bonded on one side within an infinite plate was obtained by using crack-bridging model and weight function (Kim et al., 2000; Kim and Lee, 2000).

This paper described the calculation of the stress intensity factor for a partially patched center crack within a finite plate using its approximate weight function. Additionally, the stress intensity factors for a single or double cracks initiated from notch, as often shown in the failure of aircraft components, were also obtained using its approximate weight function.

2. Definition of Problem: A Partially Patched Crack

The problem considered here is a finite center-cracked plate, with a crack of length $2a$ and under a remote uniform tensile traction σ_∞ as shown in Fig. 1. The crack is partially patched on one side with a composite material. Subscripts, P, R and A are used to identify parameters corresponding to the plate, the reinforcing patch and the adhesive layer, respectively. Thus E_P and E_R denote the Young's modulus of the plate and the reinforcement, G_A for the shear modulus of the adhesive, and t_P, t_R, t_A being the respective thickness. Here some assumptions are made; (i) the plate and the reinforcement are both isotropic and have the same Poisson ratio $\nu(=\nu_P=\nu_R)$ and all deformations are linearly elastic, (ii) there is no out-of-plane bending due to the one-sided reinforcement and no residual thermal stress induced by bonding process, (iii) the reinforced plate has no variation across the thickness. The crack length a is assumed as a small value compared to the height of the patch, B. For the cross-section configuration shown in Fig. 1(b), the redistribution of stress in a un-cracked plate can be explicitly using the one-dimensional theory of bonded joints (Rose, 1988). The reduced stress is expressed as $\sigma_0=\sigma_\infty/(1+S)$ where $S=E'_R t_R/E'_P t_P$ represents the ratio of the plate and the reinforcement stiffness. Here $E'=E/(1-\nu^2)$ is Young's modulus for the plane strain condition and $E'=E$ for the plane stress condition. Then it can be assumed that distributed linear springs act between the crack faces as shown in Fig. 2 and the boundary conditions are described as follows:

for $|x|\leq b$

$$\sigma_o(x)=\sigma_\infty,\ \text{as}\ x^2+y^2\rightarrow\infty \tag{1}$$

$$\sigma_{yy}=0,\ \text{at}\ |x|<a,\ y=0 \tag{2}$$

and for $b<|x|\leq a$

$$\sigma_o(x)=\sigma_0,\ \text{as}\ x^2+y^2\rightarrow\infty \tag{3}$$

$$\sigma_o(x)=kE'_P u_y(x),\ \text{at}\ y=0 \tag{4}$$

where k is the constraint factor. Under the plane stress condition, the appropriate value of k can be determined from the stress-displacement relation

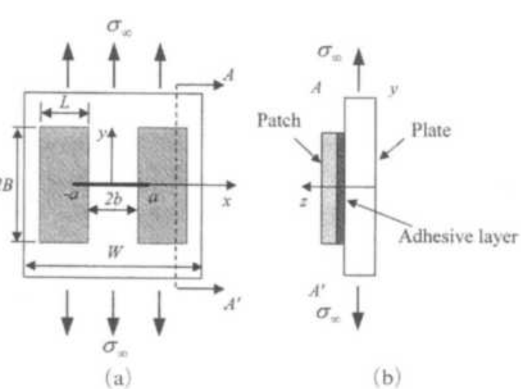

Fig. 1 Repair configuration: (a) Partially patched crack, (b) Cross-section along A-A'

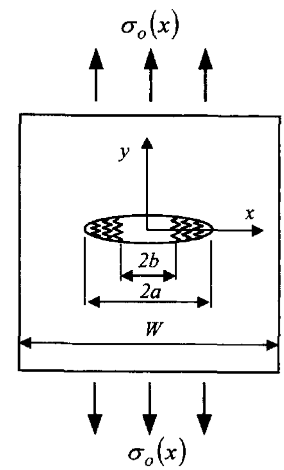

Fig. 2 Distributed springs model for a partially patched crack

for the overlap shear joint shown in Fig. 3 (Rose, 1988). The displacement can be represented as

$$u_P=\frac{\sigma_0 t_P t_A \beta}{G_A} \tag{5}$$

where β is denoted as

$$\beta=\sqrt{\frac{G_A}{t_A}\left(\frac{1}{E_P t_P}+\frac{1}{E_R t_R}\right)}=\sqrt{\frac{G_A}{t_P t_A E_P}}\left(1+\frac{1}{S}\right) \tag{6}$$

The constraint factor is defined as $1/\pi\Lambda$ with the characteristic length Λ and calculated using Eqs. (4) and (5) as

$$g=\frac{1}{\pi\Lambda}=\frac{\sigma_0}{E_P u_P}=\frac{G_A}{E_P t_P t_A \beta} \tag{7}$$

3. Calculation of Stress Intensity Factor of A Patched Crack using Approximate Weight Function

The crack surface displacement $u_o(x)$ under a remote applied stress $\sigma_o(x)$ and the crack surface displacement $u_S(x)$ due to stresses $\sigma_S(x)$ exerted on distributed springs between the crack surface can be calculated by using the weight function $m(x, a)$ (Kim, 2000). Thus the resultant crack surface displacement $u_y(x)$ at x is expressed as

$$\begin{aligned} u_y(x) &= u_0(x)-u_S(x) \\ &= \frac{1}{E_P}\int_x^a\left[\int_0^a \sigma_0(x)\, m(x, a)\, dx\right] m(x, a)\, da \\ &\quad -\frac{1}{E_P}\int_x^a\left[\int_0^a \sigma_S(x)\, m(x, a)\, dx\right] m(x, a)\, da \end{aligned} \tag{8}$$

On the other hand, the crack surface displacement at x_i due to a uniform stress σ_j acting on a

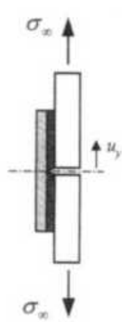

Fig. 3 Cross-section for overlap shear joint subjected to loading

segment $2w$ of the crack surface located at x_j can be expressed as

$$u(x_i, x_j)=\frac{1}{E_P}\int_{x_i}^a\left[\int_{x_j-w}^{x_j+w} \sigma_j m(x, a)\, dx\right] m(x_i, a)\, da \tag{9}$$

The influence function is defined as $g(x_i, x_j)=u(x_i, x_j)/\sigma_j$ using Eq. (9). Thus, the crack surface displacement at x_i for uniform stresses acting on the n segments corresponding to the distributed springs is simply expressed as

$$u_S(x_i)=\sum_{j=1}^n \sigma_j g(x_i, x_j) \tag{10}$$

where σ_j is given as $E_P k u_y(x_j)$ defined in Eq. (4). By substituting Eq. (10) into Eq. (8), the unknown crack surface displacement $u_y(x_i)$ located at x_i is finally obtained as

$$\begin{aligned} u_y(x_i) &= u_0(x_i)-\sum_{j=1}^n \sigma_j \sigma(x_i, x_j) \\ &= u_0(x_i)-\sum_{j=1}^n E_P k u_y(x_j)\, g(x_i, x_j) \end{aligned} \tag{11}$$

Expressing the influence function $g(x_i, x_j)$ as g_{ij}, the numerical solution for the linear system of Eq. (11) can be obtained from the Gauss-Seidel iterative method (Newman, 1982). Using the crack surface displacement obtained, the stress intensity factor $K_r(a)$ for the patched crack is calculated as

$$\begin{aligned} K_r(a) &= \int_0^a \sigma_0(x)\, m(x, a)\, dx \\ &\quad -\sum_{i=1}^n \int_{x_i-w}^{x_i+w} E_P k u_y(x_i)\, m(x, a)\, dx \end{aligned} \tag{12}$$

In case of a cracked plate with the finite width, W, as shown in Fig. 1(a), the stress intensity factor can be calculated by using the corresponding approximate weight function which is obtained from derivative of the following approximate crack surface displacement (Wang, 1991):

$$u_r(x, a)=\sqrt{8}\,\frac{\sigma_r}{E}\, a f_r(a) \sum_{i=0}^n C_i\left(1-\frac{x}{a}\right)^{i+(1/2)} \tag{13}$$

where x is the coordinate of crack surface, σ_r being the normalized stress, and $f_r(a)$ is a normalized stress intensity factor that means the geometric factor for a crack subjected to σ_r loading. And the variable C_i is obtained from the boundary conditions of the corresponding crack (Wang, 1991).

4. Calculation of Stress Intensity Factor for A Partially Patched Crack

The dimensions and material properties of the cracked plate, reinforcement and the adhesive layer are summarized in Table 1 (Wang and Rose, 1998). The dimensions, W, L, b shown in Fig. 1(a) were 120 mm, 30 mm and 30 mm, respectively. The width, w, of segment used in Eq. (9) was 0.05 mm. Figure 4 shows the stress intensity factor for a partially patched center crack within a finite plate or an infinite plate. Here $K_{o,\,finite}$ is the stress intensity factor due to the only stress $\sigma_0(x)$ reduced by reinforcement layer without considering the distribution of springs (crack-bridging model) while $K_{r,\,finite}$ is the stress intensity factor obtained from Eq. (12) with considering the reduction stress and the distribution of springs. In case of $2a/W \leq 0.5$, the

Table 1 Physical dimensions and material properties of a typical repair (Wang and Rose, 1988)

Layer	E(GPa)	ν	Thickness (mm)
Plate	71	0.3	3.0
Reinforcement	207	0.3	1.02
Adhesive	0.7*	0.33	0.203

* Adhesive's shear modulus G_A

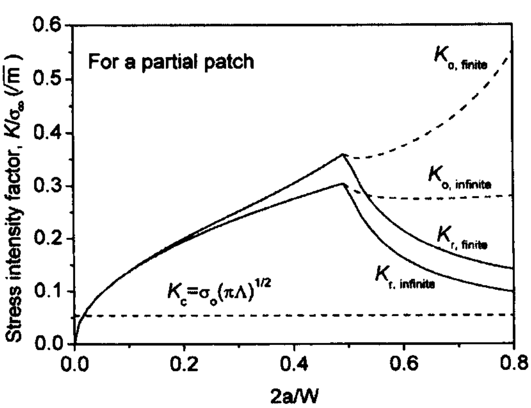

Fig. 4 Stress intensity factor for a partially patched center crack

stress intensity factors are identical because the reduced stress, $\sigma_0(x)$, and the remote applied stress, σ_∞, are the same. In the case of $2a/W > 0.5$, however, the stress intensity factors are different due to the effect of distribution of springs as shown in Fig. 2 by attaching the patch for reinforcement. The approximate weight function of Eq. (13) was used to calculate the stress intensity factor. On the other hand, in case of a center crack within an infinite plate, the stress intensity factors, $K_{o,\,infinite}$ and $K_{r,\,infinite}$ were obtained from the following the weight function (Rice, 1972):

$$m(x,\ a) = \sqrt{\frac{a}{\pi}} \frac{1}{a^2 - x^2} \tag{14}$$

The stress intensity factors for a center crack within a finite plate were larger than those of an infinite plate. Especially, in case of a patched crack, it was found that the stress intensity factor was significantly influenced by the distribution of springs acting along the crack surfaces. In addition, the stress intensity factors approach to an upper bound value, $K_c = \sigma_o\sqrt{\pi\Lambda}$, which corresponds to the stress intensity factor for a fully patched center crack within an infinite plate (Rose, 1988).

Figure 5 shows the infinite or finite cracked plates subjected to σ_∞ with a crack initiated from a circular or a semi-circular notche. The crack length, a, was assumed as a small value compared to the dimensions of the reinforcement plate. The stress intensity factor for a cracked plate shown in Fig. 5(a) was calculated by using the approximate weight function obtained from Eq. (13), which needs the following normalized stress and normalized stress intensity factor (Kujawski, 1991):

$$\frac{\sigma_i(x/\rho)}{\sigma_\infty} = 1 + 0.5\left(1 + \frac{x}{\rho}\right)^{-2} + 1.5\left(1 + \frac{x}{\rho}\right)^{-4} \tag{15}$$

$$\begin{aligned} f_r\left(\frac{a}{\rho}\right) &= \frac{K}{\sigma_\infty\sqrt{\pi a}} \\ &= 1.683f\left[\left(1 + 2\frac{a}{\rho}\right)^{-0.5} + \left(1 + 2\frac{a}{\rho}\right)^{-1.5}\right]\sqrt{\frac{a}{\rho}} \end{aligned} \tag{16}$$

where ρ is the radius of curvature, and

$$f=1 \qquad \text{for } \rho/a<a.2$$

$$f=1+0.206\left(\frac{a}{\rho}-0.2\right) \quad \text{for } \rho/a\geq 0.2$$

The normalized stress and normalized stress intensity factor for Fig. 5(b) can be represented as (Newman, 1994)

for $\alpha<1.5$

$$\frac{\sigma_r(\xi)}{\sigma_\infty}=3.1635-6.9765\xi+14.1306\xi^2-19.217\xi^3 +16.85\xi^4-8.9712\xi^5+2.616\xi^6-0.318\xi^7 \tag{17}$$

$$f_r(\alpha)=\frac{K}{\sigma_\infty\sqrt{\pi a}} =3.5479-7.2009\alpha+15.7223\alpha^2-22.466\alpha^3 +20.0387\alpha^4-10.529\alpha^5+2.96\alpha^6-0.342\alpha^7 \tag{18}$$

where ξ is x/ρ for α being a/ρ.

In a similar way shown in Fig. 2, it was assumed that the springs were distributed fully along the crack surfaces. In the cracked plates shown in the Fig. 5, therefore, the reduced stress, $\sigma_0(x)$, can be expressed by $\sigma_r(x/\rho)/(1+S)$. Figure 6 represents the stress intensity factors for double cracks initiated from a circular hole within an infinite plate. The radius of curvature, ρ, is 10 mm. Here K_∞ is the stress intensity factor without the patch for reinforcement, K_0 for only the reduced stress due to the reinforcement plate, and K_r for stress intensity factor obtained from Eq. (12) considering the distribution of spring along crack surface. When the crack length, a, is very small, the stress intensity factor, K_r, is similar to K_0 because the reduced stress, σ_0, is more dominant than the effect of spring distribution along the crack surfaces. However, as the crack length increases, the stress intensity factor K_r approaches an upper bound value (Rose, 1988), $K_c=\sigma_0\sqrt{\pi\Lambda}$. Figure 7 presents the stress intensity factors for a single crack initiated from a semi-circular notch. Also the stress intensity factors, K_∞, K_0 and K_r are shown, and the radius of curvature is 10 mm. The approximate weight function was obtained from Eqs. (13) and (14). As shown in Fig. 6, the stress intensity factor, K_r,

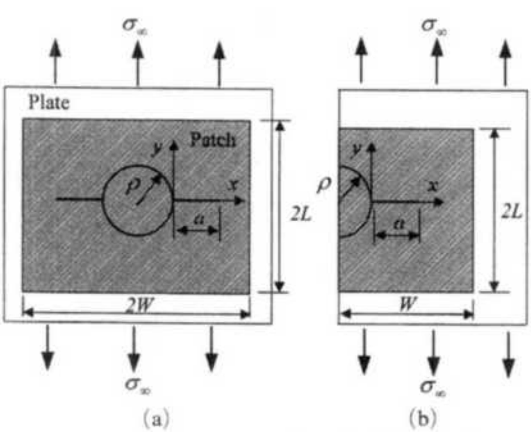

Fig. 5 Schematic of patched cracks initiated from a notch : (a) double cracks from a hole ; (b) a single crack from a single edge notch

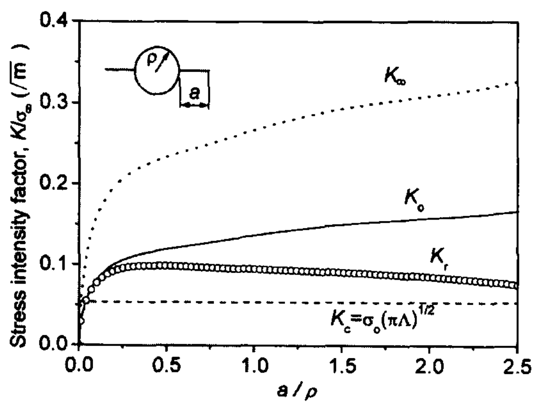

Fig. 6 Stress intensity factor for double fully patched cracks initiated from a hole

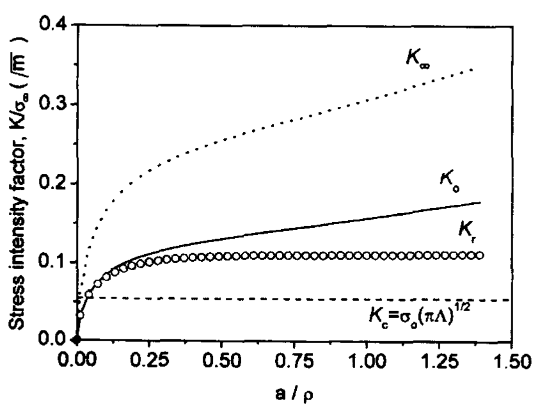

Fig. 7 Stress intensity factor for a fully patched crack initiated from a single semi-circular notch

is similar to K_0 when the crack length is small while the stress intensity factor decreases with the crack length. However, the stress intensity factor decreases more slowly than that of double cracks initiated from a circle hole. This indicates the effect of the reinforcement for double cracks within the patched plate is more effective than that of a single crack at the semi-circular notch.

5. Conclusions

The stress intensity factor for a partially patched center crack within a finite plate was obtained successfully by using an approximate weight function and the crack-bridging model. Additionally, in case of a single or double cracks initiated from a notch, the stress intensity factors considering the effect of the reinforcement plate could be easily calculated by using its approximate weight function.

This crack-bridging model using weight function shows the capability of calculating the stress intensity factor for a patched crack subjected to an arbitrary loading.

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